Regression

Recap

- Correlations are their own effect size
- On a scale of -1 to 1
- Useful for depicting relationships

Today

Regression

- What is it? Why is it useful
- Nuts and bolts
  - Equation
  - Ordinary least squares
  - Interpretation

Regression

- Regression is an umbrella term -- lots of things fall under "regression"
- This system can handle a variety of forms of relations, although all forms have to be specified in a *linear* way.

The output of regression includes both effect sizes and statistical significance. We can also incorporate multiple influences (IVs) and account for their intercorrelations.

Regression

- **Scientific** use: explaining the influence of one or more variables on some outcome.
    - Does this intervention affect reaction time?
    - Does self-esteem predict relationship quality?
- **Prediction** use: We can develop models based on what's happened in the past to predict what will happen in the figure.
    - Insurance premiums
    - Graduate school... success?
- **Adjustment**: Statistically control for known effects
    - If everyone had the same level of SES, would abuse still be associated with criminal behavior?

How does Y vary with X?

- The regression of Y (DV) on X (IV) corresponds to the line that gives the mean value of Y corresponding to each possible value of X
- "Our best guess" regardless of whether our model includes categories or continuous predictor variables

Regression Equation

$$Y = b_0 + b_1 X + e$$

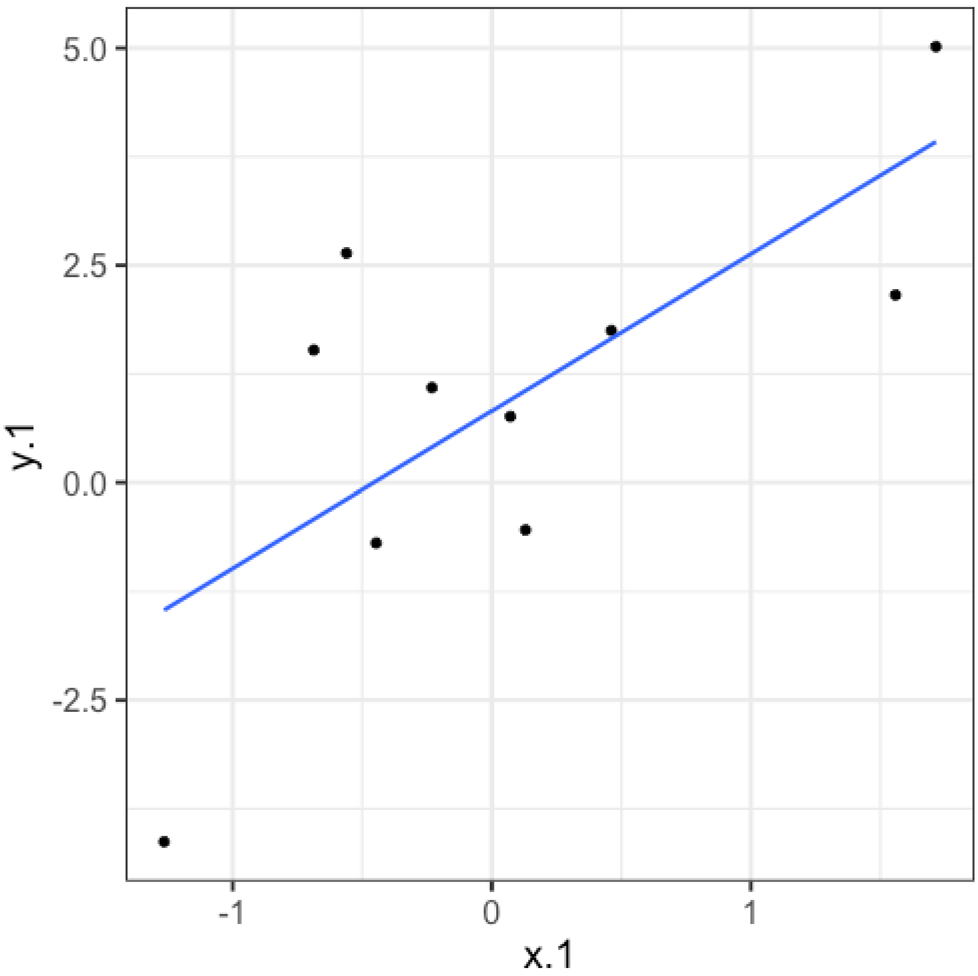
$$\hat{Y} = b_0 + b_1 X$$

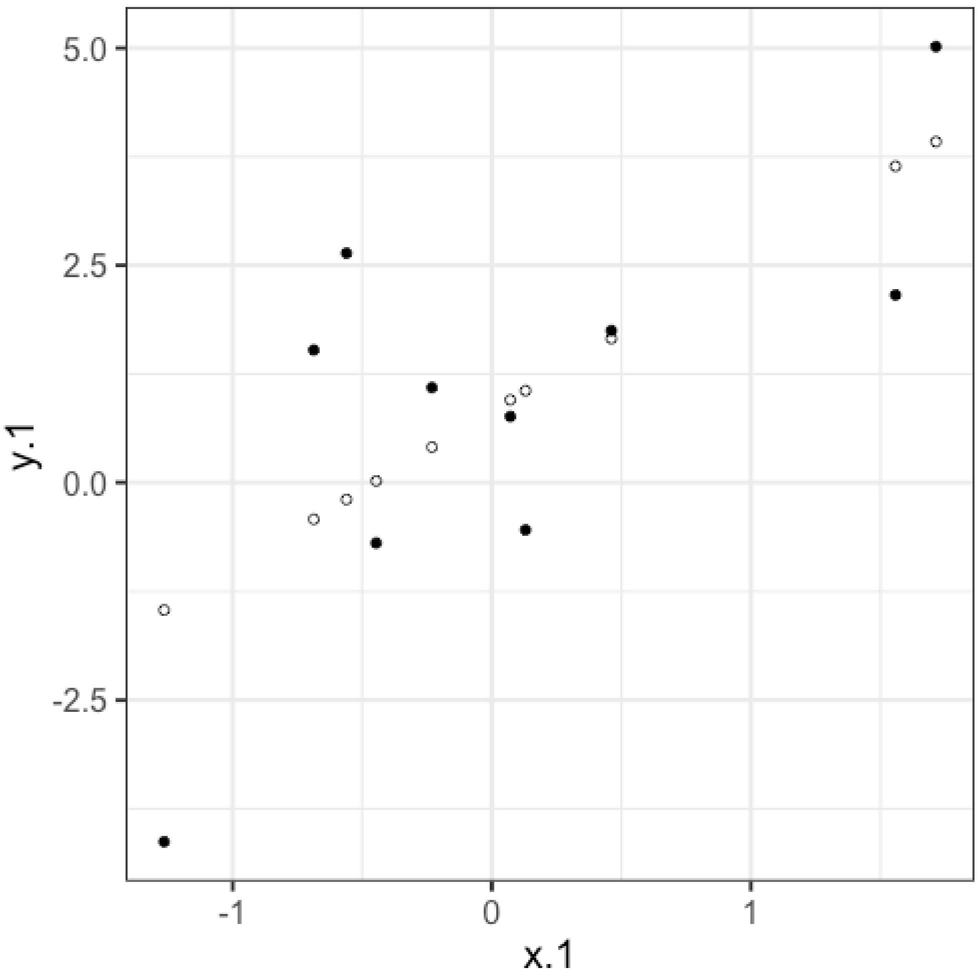
OLS

- How do we find the regression estimates?
- Ordinary Least Squares (OLS) estimation
- Minimizes deviations

$$min \sum (Y_i - \hat{Y})^2$$

- Other estimation procedures possible (and necessary in some cases)


5.0
2.5
0.0
-2.5
y.1
-1
0
1
x.1


5.0
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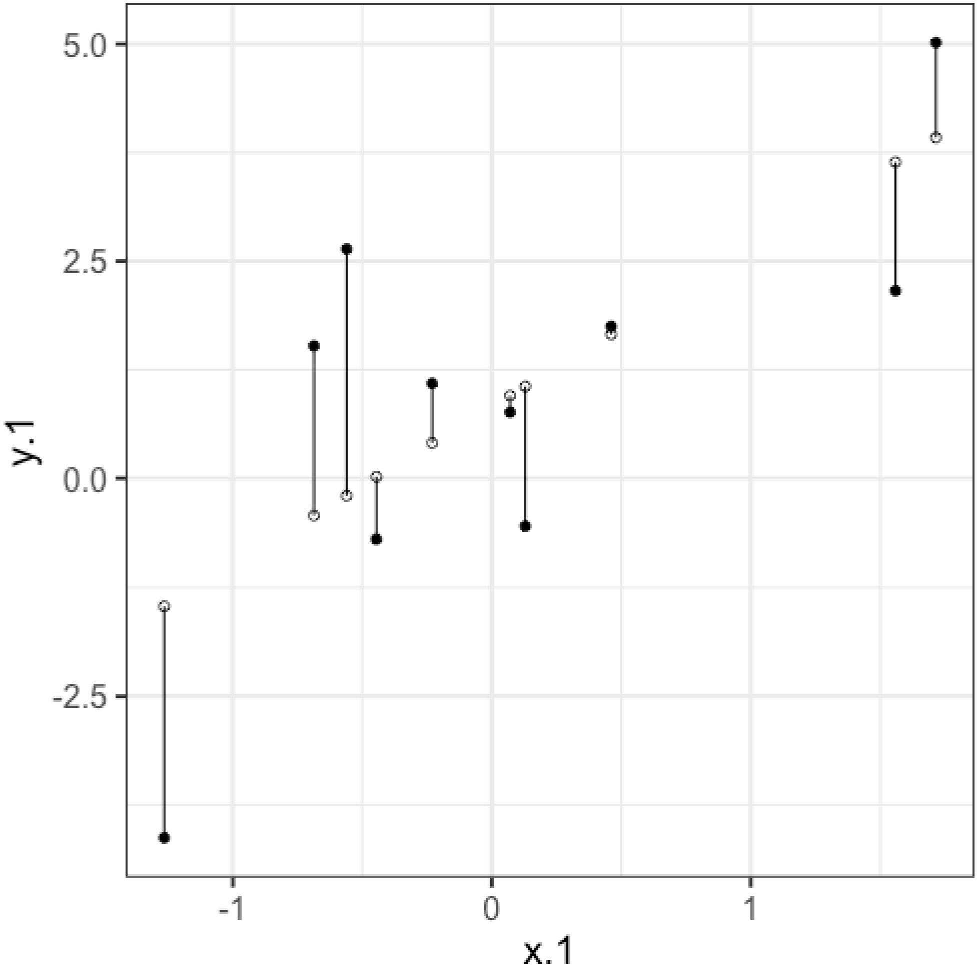
5.0
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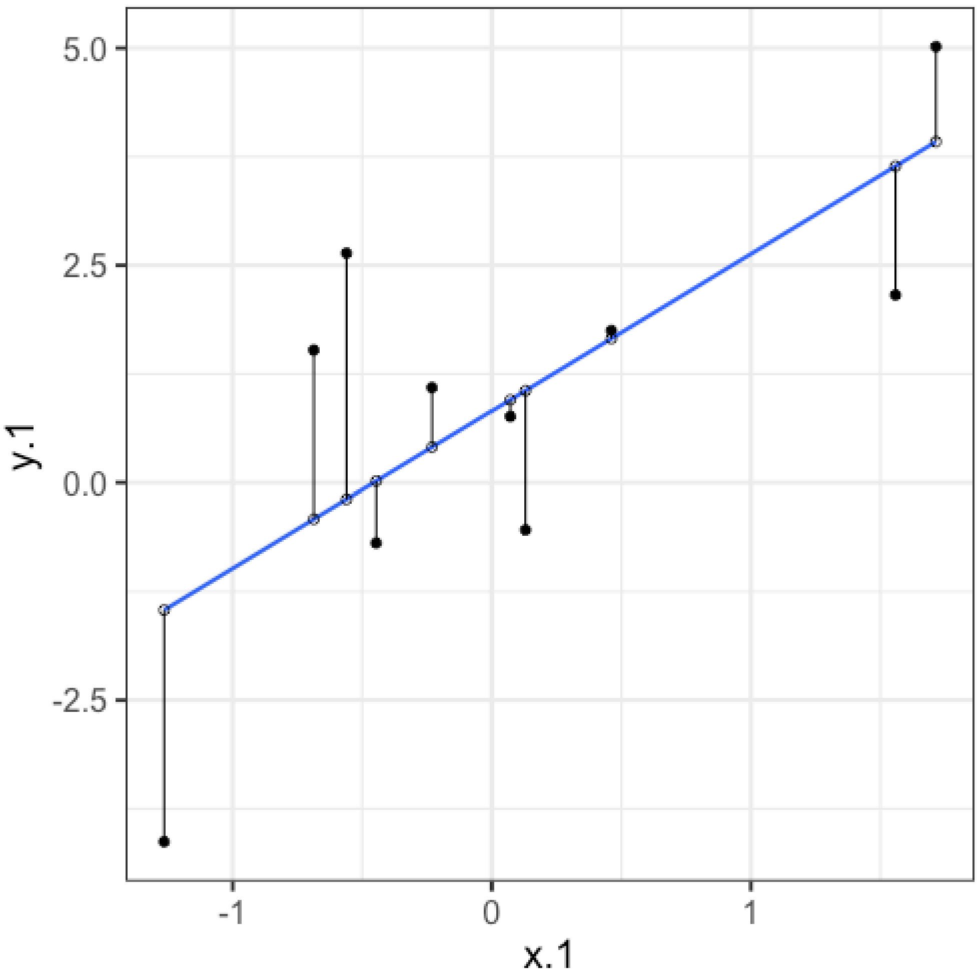
5.0
2.5
0.0
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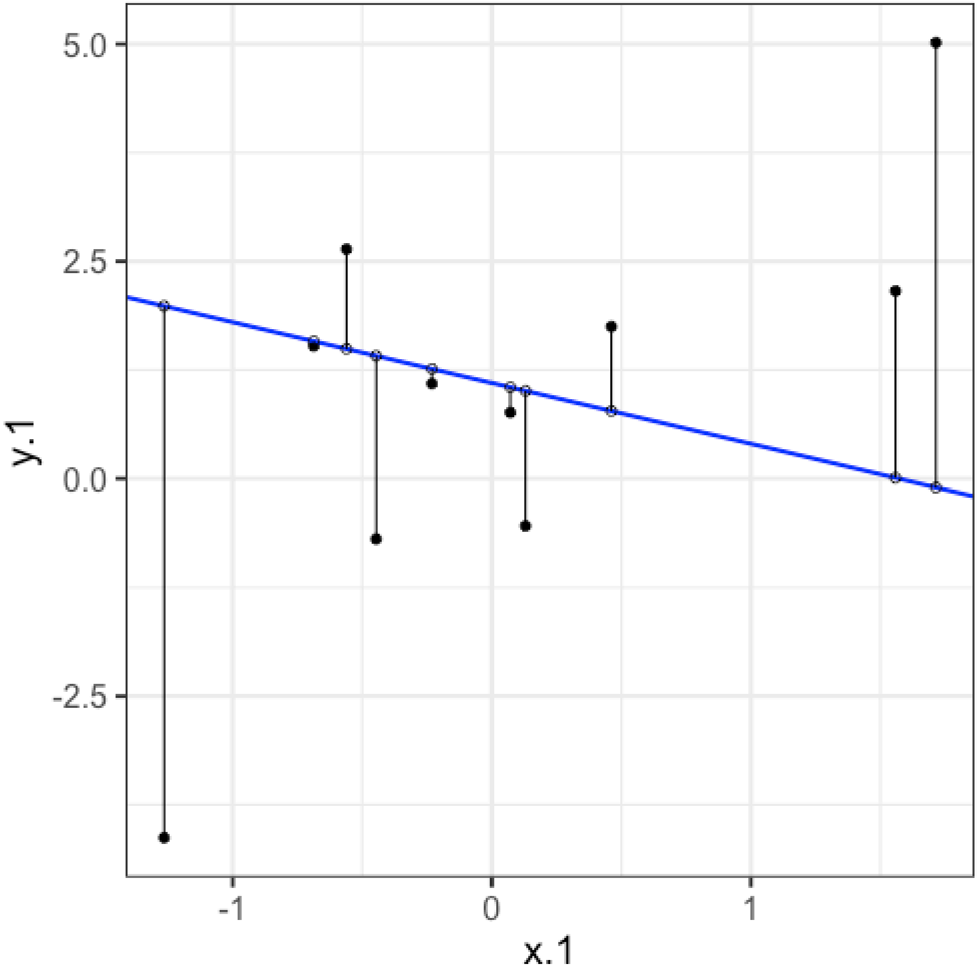
y.1
-1
0
1
x.1



compare to bad fit

$$Y = b_0 + b_1 X + e$$

$$\hat{Y} = b_0 + b_1 X$$

$$Y_i = \hat{Y}_i + e_i$$

$$e_i = Y_i - \hat{Y}_i$$

OLS

The line that yields the smallest sum of squared deviations

$$\Sigma(Y_i - \hat{Y}_i)^2$$
$$= \Sigma(Y_i - (b_0 + b_1 X_i))^2$$
$$= \Sigma(e_i)^2$$

In order to find the OLS solution, you could try many different coefficients $(b_0 \text{ and } b_1)$ until you find the one with the smallest sum squared deviation. Luckily, there are simple calculations that will yield the OLS solution every time.

In R

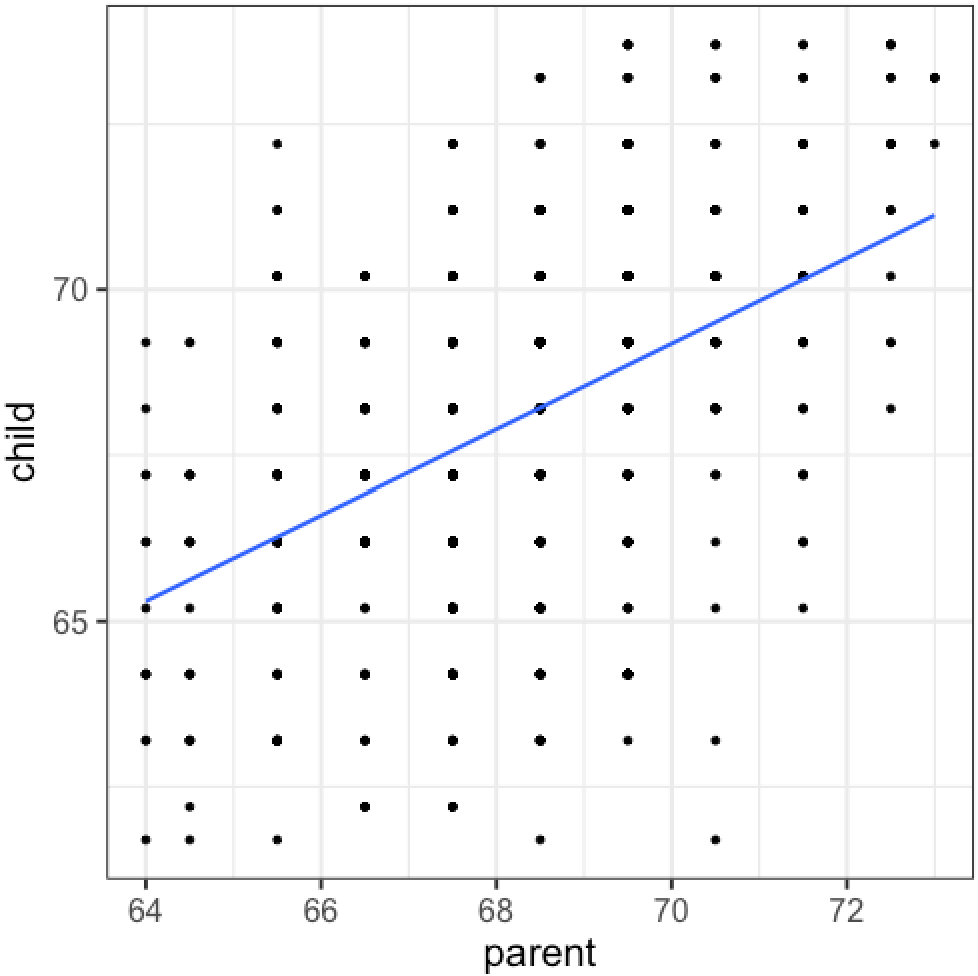
What if we regress parent height onto child height?

```
fit.1 <- lm(child ~ parent, data = galton.data)
summary(fit.1)
```

```
##
## Call:
## lm(formula = child ~ parent, data = galton.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.8050 -1.3661  0.0487  1.6339  5.9264
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 23.94153    2.81088   8.517   <2e-16 ***
## parent       0.64629    0.04114  15.711   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.239 on 926 degrees of freedom
## Multiple R-squared:  0.2105,    Adjusted R-squared:  0.2096
## F-statistic: 246.8 on 1 and 926 DF,  p-value: < 2.2e-16
```

> A 1-unit change in X predicts a *b* change in Y
>
> A 1-standard deviation change in X predicts a *b* standard deviation change in Y

Child Height Predicted By Parent Height



Data, predicted, and residuals

```r
library(broom)
model_info = augment(fit.1)
head(model_info)
```

```
## # A tibble: 6 x 8
##   child parent .fitted .resid .std.resid    .hat .sigma .cooksd
##   <dbl>  <dbl>   <dbl>  <dbl>      <dbl>   <dbl>  <dbl>   <dbl>
## 1  61.7   70.5    69.5  -7.81      -3.49 0.00270   2.22 0.0165
## 2  61.7   68.5    68.2  -6.51      -2.91 0.00109   2.23 0.00462
## 3  61.7   65.5    66.3  -4.57      -2.05 0.00374   2.23 0.00787
## 4  61.7   64.5    65.6  -3.93      -1.76 0.00597   2.24 0.00931
## 5  61.7   64      65.3  -3.60      -1.62 0.00735   2.24 0.00966
## 6  62.2   67.5    67.6  -5.37      -2.40 0.00130   2.23 0.00374
```

```r
describe(model_info)
```

```
##              vars   n  mean   sd median trimmed  mad   min   max range  skew
## child           1 928 68.09 2.52  68.20   68.12 2.97 61.70 73.70 12.00 -0.09
## parent          2 928 68.31 1.79  68.50   68.32 1.48 64.00 73.00  9.00 -0.04
## .fitted         3 928 68.09 1.16  68.21   68.10 0.96 65.30 71.12  5.82 -0.04
## .resid          4 928  0.00 2.24   0.05    0.06 2.26 -7.81  5.93 13.73 -0.24
## .std.resid      5 928  0.00 1.00   0.02    0.03 1.01 -3.49  2.65  6.14 -0.24
## .hat            6 928  0.00 0.00   0.00    0.00 0.00  0.00  0.01  0.01  1.99
```

Are we doing a good job?

- The way the world is = our model + error
- How good is our model? Does it "fit" the data well?

To assess how well our model fits the data, we simply take all the variability in our outcome and partition it into different categories. For now, we will partition it into two categories: the variability that is predicted by (explained by) our model, and variability that is not.

To the extent that we can generate different predicted values of Y *based on the different values of X*, we are doing well with our model.

R^2

- R^2 is the amount of variance in Y that is explained by X (aka by your model)
- measure of **model fit**; more variance explained, better your model

R^2

```
fit.1 = lm(child ~ parent, data = galton.data)
summary(fit.1)
```

```
##
## Call:
## lm(formula = child ~ parent, data = galton.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.8050 -1.3661  0.0487  1.6339  5.9264
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```

Residual Standard Error

- $Residuals = Y - \hat{Y}$
- There is a residual for each individual (each person has an observed score and a predicted score)
- You can plot these residuals. The *dispersion* or spread of these residuals is called the **Residual Standard Error (RSE)**
- The RSE is the standard deviation of all of these residuals (in original units); it is the standard deviation of Y that is **not** accounted for by the model
- If it's a fat distribution, that means the residuals are large; we're not doing great
- If it's a skinny distribution, then the residuals are smaller; we're doing a good job!

RSE

```
fit.1 = lm(child ~ parent, data = galton.data)
summary(fit.1)
```

```
##
## Call:
## lm(formula = child ~ parent, data = galton.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.8050 -1.3661  0.0487  1.6339  5.9264
##
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```
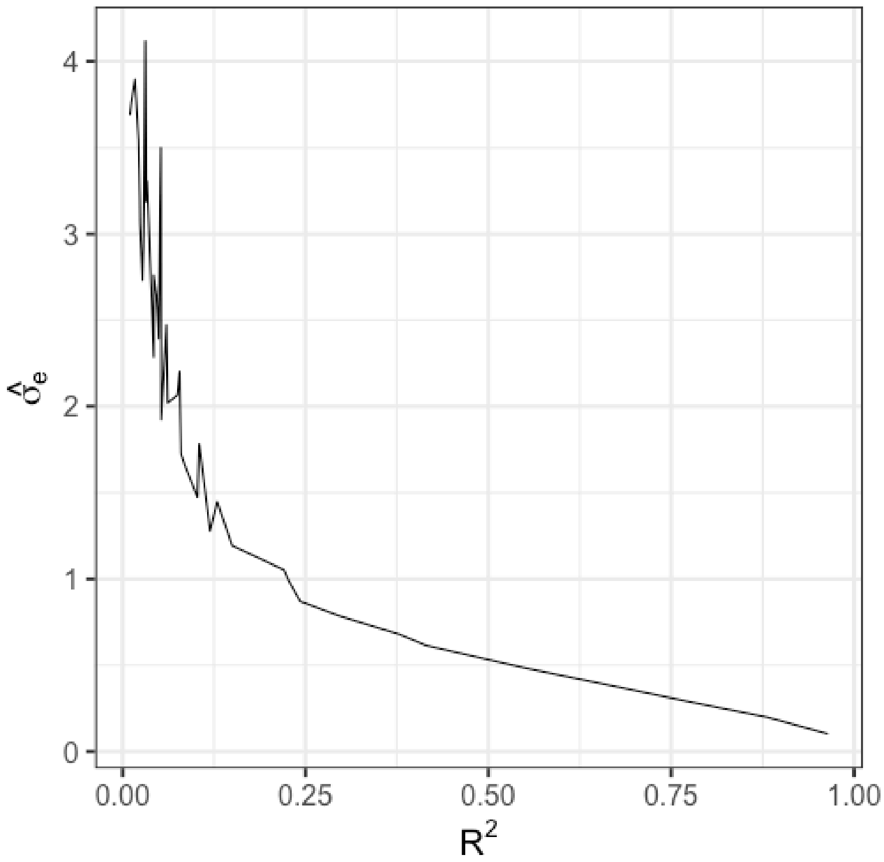

R^2 and residual standard deviation

- two sides of same coin
- one in original units, the other standardized



Inferential Tests

Omnibus test

- Test of whether the model fits the data

Regression Coefficients

- Is our predictor significant?

Omnibus test

Overall, our goal is to partition variance. We want to know if the variance explained by our model is larger than the variance that is left over or *unexplained*.

Our sampling distribution will be the F distribution. The z and the t test for differences in means. F distribution looks at the size of a **ratio of variances**. The ratio of explained to unexplained variance. The ratio of your regression to error.

Yes, this is analogous to ANOVA. But ANOVAs require categorical predictors. Regression is more flexible!

ANOVA is a special case of regression!

```r
anova(fit.1)
```

```
## Analysis of Variance Table
##
## Response: child
##            Df Sum Sq Mean Sq F value    Pr(>F)
## parent      1 1236.9 1236.93  246.84 < 2.2e-16 ***
## Residuals 926 4640.3    5.01
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

ANOVA is a special case of regression!

```
summary(fit.1)
```

```
##
## Call:
## lm(formula = child ~ parent, data = galton.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.8050 -1.3661  0.0487  1.6339  5.9264
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```

Predictors

$$H_0 : \beta_1 = 0$$
$$H_1 : \beta_1 \neq 0$$

Predictors

- Does X provide any predictive information?
- Does X provide any explanatory power regarding the variability of Y?
- Is the the average value the best guess (i.e., is Y bar equal to the predicted value of Y?)
- Is the regression line flat?
- Are X and Y correlated?

Predictors

- **One-sample t-tests**
- $t = \frac{b}{se}$

```
summary(fit.1)
```

```
##
## Call:
## lm(formula = child ~ parent, data = galton.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.8050 -1.3661  0.0487  1.6339  5.9264
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 23.94153    2.81088   8.517   <2e-16 ***
## parent       0.64629    0.04114  15.711   <2e-16 ***
## ---
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```

Coding Tips for Regression

- You either need to store the `lm` model as it's own object, and then call `summary()` on it. OR, you can nest `lm()` within the `summary()` function like: `summary(lm(child ~ parent, data = galton.data))`

```
# summary(fit.1)

summary(lm(child ~ parent, data = galton.data))
```

```
##
## Call:
## lm(formula = child ~ parent, data = galton.data)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.8050 -1.3661  0.0487  1.6339  5.9264
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```

Coding Tips for Regression

- The `broom` package is part of the `tidyverse`, but it does not load automatically; you'll need to load it separately

`tidy()` creates a data.frame from the output table

```
tidy(fit.1)
```

```
## # A tibble: 2 x 5
##   term        estimate std.error statistic  p.value
##   <chr>          <dbl>     <dbl>     <dbl>    <dbl>
## 1 (Intercept)   23.9      2.81        8.52 6.54e-17
## 2 parent         0.646    0.0411     15.7  1.73e-49
```

Coding Tips for Regression

- The `broom` package is part of the `tidyverse`, but it does not load automatically; you'll need to load it separately

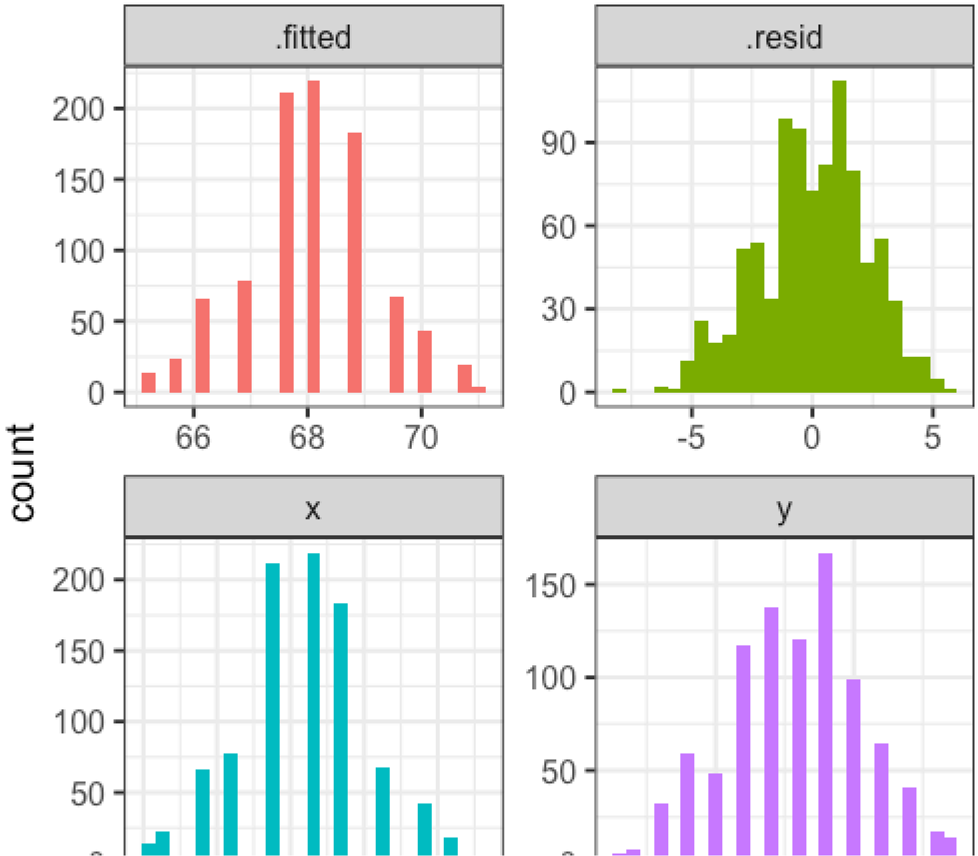
`augment` adds columns to a dataset, including things like fitted values and residuals. If it has a `.` in front of the column name, it was added. Also stored as a data.frame so we can use them later.

```
augment(fit.1)
```

```
## # A tibble: 928 x 8
##    child parent .fitted .resid .std.resid    .hat .sigma .cooksd
##    <dbl>  <dbl>   <dbl>  <dbl>      <dbl>   <dbl>  <dbl>   <dbl>
##  1  61.7   70.5    69.5  -7.81      -3.49 0.00270   2.22 0.0165
##  2  61.7   68.5    68.2  -6.51      -2.91 0.00109   2.23 0.00462
##  3  61.7   65.5    66.3  -4.57      -2.05 0.00374   2.23 0.00787
##  4  61.7   64.5    65.6  -3.93      -1.76 0.00597   2.24 0.00931
##  5  61.7   64      65.3  -3.60      -1.62 0.00735   2.24 0.00966
##  6  62.2   67.5    67.6  -5.37      -2.40 0.00130   2.23 0.00374
##  7  62.2   67.5    67.6  -5.37      -2.40 0.00130   2.23 0.00374
##  8  62.2   67.5    67.6  -5.37      -2.40 0.00130   2.23 0.00374
##  9  62.2   66.5    66.9  -4.72      -2.11 0.00218   2.23 0.00487
## 10  62.2   66.5    66.9  -4.72      -2.11 0.00218   2.23 0.00487
```

Coding Tips for Regression

`augment` adds columns to a dataset, including things like fitted values and residuals. If it has a `.` in front of the column name, it was added. Also stored as a data.frame so we can use them later.



Coding Tips for Regression

- The `broom` package is part of the `tidyverse`, but it does not load automatically; you'll need to load it separately

`glance` gives you the F-test & fit measures

```
glance(fit.1)
```

```
## # A tibble: 1 x 12
##   r.squared adj.r.squared sigma statistic  p.value    df logLik   AIC   BIC
##       <dbl>         <dbl> <dbl>     <dbl>    <dbl> <dbl>  <dbl> <dbl> <dbl>
## 1     0.210         0.210  2.24      247. 1.73e-49     1 -2064. 4133. 4148.
## # … with 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

Adding more predictors

- You can enter lots of variables into your regression; you aren't limited to just 1
- $Y = b_0 + b_1X_1 + b_2X_2 + \ldots + b_nX_n + e$
- Interpretation of b coefficients changes slightly: "a 1-unit change in X_1 predicts a __ change in Y, *while controlling for* X_2
- Interactions get even more tricky (beyond our scope, sadly)

It's. All. Regression.

Really want to up your stats games? Go through this site so you can see how it's all just actually regression.

A cheatsheet:

Common statistical tests are linear models

Last updated: 28 June, 2019.Also check out the Python version!

See worked examples and more details at the accompanying notebook: https://lindeloev.github.io/tests-as-linear

	Common name	Built-in function in R	Equivalent linear model in R	Exact?	The linear model in words	Icon
Simple regression: lm(y ~ 1 + x)	**y is independent of x** P: One-sample t-test N: Wilcoxon signed-rank	t.test(y) wilcox.test(y)	lm(y ~ 1) lm(signed_rank(y) ~ 1)	✓ for N >14	One number (intercept, i.e., the mean) predicts **y**. - (Same, but it predicts the *signed rank* of **y**.)	
	P: Paired-sample t-test N: Wilcoxon matched pairs	t.test(y_1, y_2, paired=TRUE) wilcox.test(y_1, y_2, paired=TRUE)	lm(y_2 - y_1 ~ 1) lm(signed_rank(y_2 - y_1) ~ 1)	✓ for N >14	One intercept predicts the pairwise **y_2-y_1** differences. - (Same, but it predicts the *signed rank* of **y_2-y_1**.)	
	y ~ continuous x P: Pearson correlation N: Spearman correlation	cor.test(x, y, method='Pearson') cor.test(x, y, method='Spearman')	lm(y ~ 1 + x) lm(rank(y) ~ 1 + rank(x))	✓ for N >10	One intercept plus **x** multiplied by a number (slope) predicts **y**. - (Same, but with *ranked* **x** and **y**)	
	y ~ discrete x P: Two-sample t-test P: Welch's t-test N: Mann-Whitney U	t.test(y_1, y_2, var.equal=TRUE) t.test(y_1, y_2, var.equal=FALSE) wilcox.test(y_1, y_2)	lm(y ~ 1 + G_2)^A gls(y ~ 1 + G_2, weights=…^{B})^A lm(signed_rank(y) ~ 1 + G_2)^A	✓ ✓ for N >11	An intercept for **group 1** (plus a difference if **group 2**) predicts **y**. - (Same, but with one variance *per group* instead of one common.) - (Same, but it predicts the *signed rank* of **y**.)	
Multiple regression: lm(y ~ 1 + x_1 + x_2 +...)	P: One-way ANOVA N: Kruskal-Wallis	aov(y ~ group) kruskal.test(y ~ group)	lm(y ~ 1 + G_2 + G_3 +...+ G_N)^A lm(rank(y) ~ 1 + G_2 + G_3 +...+ G_N)^A	✓ for N >11	An intercept for **group 1** (plus a difference if group ≠ 1) predicts **y**. - (Same, but it predicts the *rank* of **y**.)	
	P: One-way ANCOVA	aov(y ~ group + x)	lm(y ~ 1 + G_2 + G_3 +...+ G_N + x)^A	✓	- (Same, but plus a slope on **x**.) *Note: this is discrete AND continuous. ANCOVAs are ANOVAs with a continuous x.*	
	P: Two-way ANOVA	aov(y ~ group * sex)	lm(y ~ 1 + G_2 + G_3 +...+ G_N + S_2 + S_3 +...+ S_K + G_2*S_2 + G_3*S_3 + … + G_N*S_K)	✓	Interaction term: changing **sex** changes the **y ~ group** parameters. *Note: **$G_{2 \text{ to } N}$** is an indicator (0 or 1) for each non-intercept levels of the **group** variable. Similarly for **$S_{2 \text{ to } K}$** for sex. The first line (with G_i) is main effect of group, the second (with S_i) for sex and the third is the **group × sex** interaction. For two levels (e.g. male/female), line 2 would just be "S_2" and line 3 would be S_2 multiplied with each G_i.*	*[Coming]*
	Counts ~ discrete x N: Chi-square test	chisq.test(groupXsex_table)	**Equivalent log-linear model** glm(y ~ 1 + G_2 + G_3 + … + G_N + S_2 + S_3+ … + S_K + G_2*S_2 + G_3*S_3 +...+ G_N*S_K, family=...)^A	✓	Interaction term: (Same as Two-way ANOVA.) *Note: Run glm using the following arguments: `glm(model, family=poisson())` As linear-model, the Chi-square test is $\log(y_i) = \log(N) + \log(\alpha_i) + \log(\beta_j) + \log(\alpha_i\beta_j)$ where α_i and β_j are proportions. See more info in the accompanying notebook.*	*Same as Two-way ANOVA*
	N: Goodness of fit	chisq.test(y)	glm(y ~ 1 + G_2 + G_3 +...+ G_N, family=...)^A	✓	(Same as One-way ANOVA and see Chi-Square note.)	*1W-ANOVA*

Next Time

Principles behind data visualizations

Get ready for some rants…